Calculations of Fringing Fields of a Quadrupole Doublet

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Abstract—**Fringing fields of a quadrupole doublet consisting of circular concave electrodes are calculated by solving the three-dimensional Laplace equation numerically. It is found that the R and θ components in cylindrical coordinates have negligible influence on the normalized fringing potential distributions within the available lens region.**

Quadrupole lens systems have had very wide application [1] in recent years. The system usually consists of a sequence of two or more lenses. To design the lens systems precisely, it is most important to know potential distributions along the beam axis (Z axis) in the entire lens region. In the magnetic case, the distributions have been measured [2].

In this letter, an electrostatic quadrupole doublet consisting of a sequence of two lenses is considered and is analyzed by solving the three-dimensional Laplace equation numerically. Each lens consists of four quarter-circular concave electrodes as shown in Fig. 1. When lens potentials are applied to these electrodes as shown in Fig. 1(b), almost rectangular hyperbolic equipotentials are formed in the aperture plane. The gap between each electrode is chosen ($\gamma=\pi/6$) such that the equipotentials are in satisfactory agreement with rectangular hyperbolas within the available aperture region [3]. Each electrode has a mechanical length $2l_1$ and they are placed in the region $-Z_1 \leqq Z \leqq Z_1$ and $Z_2 \leqq Z \leqq 2Z_s - Z_2$, respectively. Therefore two planes normal to the Z axis at $Z=0$ and $Z=Z_s$ are the symmetrical planes of each lens.

In the numerical calculations, the following potential distributions are imposed as boundary conditions.

1) Potential distribution in the plane at $Z=0$, $\phi(R, \theta, 0)$.
2) Potential distribution in the plane at $Z=Z_s$, $\phi(R, \theta, Z_s)$.
3) Potential distribution at the surface $R=a$ within a region $0 \leqq Z \leqq Z_1$, $\phi(a, \theta, 0 \leqq Z \leqq Z_1)$.
4) Potential distribution at the surface $R=a$ within a region $Z_1 \leqq Z \leqq Z_2$, $\phi(a, \theta, Z_1 \leqq Z \leqq Z_2)$.
5) Potential distribution at the surface $R=a$ within a region $Z_2 \leqq Z \leqq Z_s$, $\phi(a, \theta, Z_2 \leqq Z \leqq Z_s)$.

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The first and second potential distributions are calculated by solving the two-dimensional Laplace equation with the boundary condition shown in Fig. 1(c).

$$\phi(R, \theta, Z_s) = c \cdot \phi(R, \theta, 0), \tag{1}$$

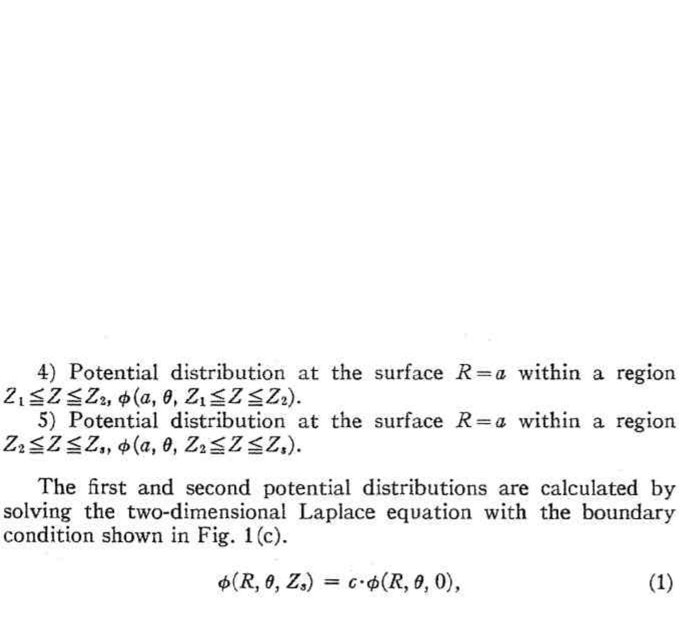

Fig. 1. Arrangement of an electrostatic quadrupole doublet. (a) Construction of the lens. (b) Cross section of the lens. (c) Potential distribution at the surface $R=a$ with a region $0 \leqq Z \leqq Z_1$.

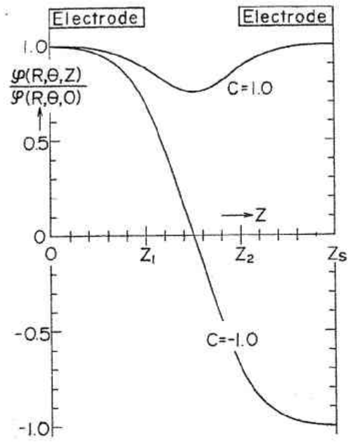

Fig. 2. Normalized potential distribution in axial direction for parameters of $b/a=0.48$, $m=0.25$, $a=l_1=S$, $R/a=0.3$, $\theta=0°$, and $c=\pm 1.0$.

where

$$c = \phi(a, \theta, Z_2 \leqq Z \leqq Z_s)/\phi(a, \theta, 0 \leqq Z \leqq Z_1). \tag{2}$$

The third potential distribution is given directly in Fig. 1(c). The fourth is assumed to be

$$\phi(a, \theta, Z_1 \leqq Z \leqq Z_2) = \left(\frac{S+l_1-Z}{S}\right)^m \cdot \phi(a, \theta, 0 \leqq Z \leqq Z_1)/\exp\left[(Z-l_1)/b\right] + \left(\frac{Z-l_1}{S}\right)^m \cdot \phi(a, \theta, Z_2 \leqq Z \leqq Z_s)/\exp\left[-(Z-l_1-S)/b\right], \tag{3}$$

where b is chosen such that $b/a=0.48$ as a result of consideration of a quadrupole singlet [4], and m is determined from experimental results [5]. The first term on the right-hand side in (3) describes the contribution to the potential at the boundary by the lens centered at $Z=0$ and the second term describes the effect of the lens centered at $Z=Z_s$. The final potential distribution is also given by (2).

Liebmann's accelerating method is used in this computer analysis. A result for the normalized fringing potential distributions is shown in Fig. 2 for $a=l_1=S$, $R/a=0.3$, $\theta=0°$, $m=0.25$, and $c=1.0$ and -1.0. The numbers of mesh points are 10, 48, and 15 for R, θ, and Z coordinates, respectively. Table I shows the results for some combinations of parameters of R/a and θ with the same lens mentioned previously. It is found from these results that the R component has only small effect on the normalized fringing potential distributions within the available aperture region ($R/a \lesssim 0.5$) and that the θ component has negligible influence on them. This has been verified experimentally [2].

TABLE I

NORMALIZED POTENTIAL DISTRIBUTION $\phi(R, \theta, Z)/\phi(R, \theta, 0)$ IN A PLANE NORMAL TO Z AXIS AT $Z=Z_1$ FOR SOME VALUES OF R/a AND θ WITH PARAMETERS OF $m=0.25$ AND $c=\pm 1.0$

θ \ R/a	0.2	0.3	0.5
c = 1.0			
0.0°	0.88649	0.88805	0.89494
15.0°	0.88652	0.88808	0.89500
37.5°	0.88668	0.88817	0.89497
c = -1.0			
0.0°	0.69211	0.70013	0.73036
15.0°	0.69214	0.70016	0.73051
37.5°	0.69232	0.70023	0.73031

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